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This paper contains 3 printed pages

Unique Paper Code : 42357602

Name of the Paper : DSE- Probability and Statistics

Name of the Course : CBCS-LOCF: B.SC. Physical Sciences/Mathematical Sciences

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your roll number on the top immediately on receipt of this question paper.
2. Attempt all the six questions.
3. Each question has three parts. Attempt any twp parts from each question.
4. Each part in Question 1, 3, 5 carries 6 marks.
5. Each part in Question 2, 4, 6 carries 6.5 marks.
6. Use of scientific calculator is allowed.

1. a) i) If A and B are events in the sample space S, then prove that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

ii) Let X be a random variable with Distribution Function F_X. Prove that for $a < b$, $P[a < X \leq b] = F_X(b) - F_X(a)$.

b) Show that $f(x) = \begin{cases} e^{-x} & ; 0 < x < \infty \\ 0 & ; \text{elsewhere} \end{cases}$ represents a probability density function. Also calculate P(X > 1)

c) i) Show that if X is a random variable and a, b are constants, then

$$E[(aX + b)^n] = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i E(X^{n-i}).$$

ii) Let the probability mass function $P(x)$ be positive at $x = -1, 0, 1$ and zero elsewhere. If $P(0) = \frac{1}{4}$, find $E[X^2]$.

2. a) The random variable X has the probability distribution :

$f(x) = \frac{1}{8}\binom{3}{x}$ $for\ x = 0, 1, 2\ and\ 3.$

Find the moment generating function of this random variable and use it to determine μ_1' and μ_2', where $\mu_r' = E[X^r]$ is the rth moment about the origin.

b) Show that the mean and the variance of the binomial distribution are $\mu = n\theta$ and $\sigma^2 = n\theta(1-\theta)$.

c) Find the probabilities that a random variable having the standard normal distribution will take on a value:

i) less than 1.30.

ii) less than −0.25.

iii) between 0.45 and 1.30.

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3. a) Show that the normal distribution has:

i) a relative maximum at $x = \mu$.

ii) inflection points at $x = \mu - \sigma$ and $x = \mu + \sigma$.

b) Let $f(x_1, x_2) = \begin{cases} 4x_1x_2 & ; \ 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & ; \ otherwise \end{cases}$ be the pdf of X_1 and X_2.

Find $P(0 < X_1 < \frac{1}{2}, \frac{1}{4} < X_2 < 1)$ and $P(X_1 < X_2)$.

c) Let the random variable X and Y have joint probability mass function as:

(x, y)	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$
$P(x, y)$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$

i) Find marginal probability mass function of X and Y.

ii) $P(X + Y \leq 2)$.

4. a) Let X_1 and X_2 have the joint pdf $f(x_1, x_2) = \begin{cases} 2 & ; 0 < x_1 < x_2 < 1 \\ 0 & ; otherwise \end{cases}$

i) Find conditional pdf $f_{1|2}(x_1|x_2)$.

ii) Find conditional mean $E(X_1|x_2)$ and the conditional variance $Var(X_1|x_2)$.

b) Let $f(x/y) = \begin{cases} \frac{cx}{y^2} & ; \ 0 < x < y, 0 < y < 1 \\ 0 & ; \ elsewhere \end{cases}$

be the conditional density of (X, Y) and

$f_2(y) = \begin{cases} ky & ; \ 0 < y < 1 \\ 0 & ; \ elsewhere \end{cases}$ be the marginal pdf of Y. Determine:

i) the constants c and k.

ii) joint pdf of X and Y.

iii) $P\left[\frac{1}{4} < X < \frac{1}{2} \middle| Y = \frac{5}{8}\right]$

iv) $P\left[\frac{1}{4} < X < \frac{1}{2}\right]$

c) Consider a random experiment that consists of drawing at random one chip from a bowl containing 10 chips of the same shape and size. Each chip has an ordered pair of numbers on it: one with (1, 1), one with (2, 1), two with (3, 1), one with (1, 2), two with (2, 2), and three with (3, 2). Let the random variables X_1 and X_2 be defined as the respective first and second values of the ordered pair. Find the joint probability mass function $p(x_1, x_2)$ of X_1 and X_2, provided with $p(x_1, x_2)$ equal to zero elsewhere.

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5. a) If two random variables X and Y have the joint density given by

$$f(x,y) = \begin{cases} e^{-x-y} & ; \quad x > 0, y > 0 \\ 0 & ; \quad elsewhere \end{cases}$$

Show that $M(t_1, t_2) = (1-t_1)^{-1}(1-t_2)^{-1}; t_1, t_2 < 1$.

Also show that $[e^{tx+ty}] = (1-t)^{-2}; t < 1$.

b) Using method of least squares to fit a straight line for the following data:

X	1	2	3	4	5
Y	5	7	9	10	11

c) If X and Y have Joint pdf-

$$f(x,y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0 ; & otherwise \end{cases}$$

Find the correlation coefficient between X and Y.

6. a) If X and Y have Joint pdf

$$f(x,y) = \begin{cases} 3x & ; 0 < y < x, \ 0 < x < 1 \\ 0 ; & otherwise \end{cases}$$

Are X and Y are independent? If not, find $E[Y|X]$.

b) If $X_i ; i = 1, 2, 3, 4, \ldots, 10$ be independent random variables, each being uniformly distributed over $(0, 1)$. Estimate $P(\sum_1^{10} X_i > 7)$.

c) A die is thrown 3600 times, show that the number of sixes lies between 550 and 650 is at least $\frac{4}{5}$.

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